

**NOTES - Inverse Functions - Switching X and Y****Part 1 - The Inverse of a Relation:**

Consider the function  $f = \{(0,3), (-6,5), (2,4)\}$ .

This function has an **inverse**, denoted as  $f^{-1}$ . The inverse is  $f^{-1} = \{(3,0), (5,-6), (4,2)\}$ .

Describe how to find the inverse: *Switch x and y*

**Part 1 Practice:**

Now find the inverse relation for  $g = \{(-4,6), (8,2), (5,0), (3,-1)\}$ .

$$g^{-1} = \{(6, -4), (2, 8), (0, 5), (-1, 3)\}$$

**Part 2 - The Inverse of a Function in Equation Form:**

You can also find an inverse for a function given in terms of  $x$ . Since the main idea above is to switch the  $x$ 's and  $y$ 's, the same principle holds for the more complicated example.

Consider  $f(x) = 4x - 5$ . To find  $f^{-1}(x)$ , just use the following steps:

1. Write  $f(x)$  as  $y$  instead.

2. Interchange  $x$  and  $y$   
(make them switch places).

3. Solve for  $y$ .

4. Write  $y$  as  $f^{-1}$ .

This is the inverse of the original function.

$$\begin{array}{l} y = 4x - 5 \\ x = 4y - 5 \\ +5 \quad +5 \end{array}$$

$$\frac{x+5}{4} = \frac{4y}{4}$$

$$y = \frac{x+5}{4}$$

$$f^{-1}(x) = \frac{x+5}{4}$$

**Part 2 Practice:**

Use the steps above to find the inverse of  $f(x) = \frac{1}{5}x + 3$ .

Step 1: Write  $f(x)$  as  $y$  instead.

Step 2: Interchange  $x$  and  $y$   
(make them switch places).

Step 3: Solve for  $y$ .

Step 4: Write  $y$  as  $f^{-1}$ .

$$\begin{array}{l} y = \frac{1}{5}x + 3 \\ x = \frac{1}{5}y + 3 \\ -3 \quad -3 \end{array}$$

$$5(x-3) = \frac{1}{5}y$$

$$f^{-1}(x) = 5x - 15$$

### Part 3 - Are They Inverses? -Verifying Your Answers:

#### With Composition

##### Option 1 - Composition

\*If two functions are inverses, then  $f[g(x)] = g[f(x)] = x$

Find  $f[g(x)]$  and  $g[f(x)]$ . If they are both equal to  $x$ , then the functions are inverses.

Practice: Use composition to test if  $f(x) = 5x + 10$  and  $g(x) = \frac{1}{5}x - 2$  are inverses.

$$f[g(x)] \\ \downarrow \\ 5\left(\frac{1}{5}x - 2\right) + 10$$

$$= x - 10 + 10$$

(X)

$$g[f(x)] \\ \downarrow \\ \frac{1}{5}(5x + 10) - 2$$

$$x + 2 - 2$$

(X)

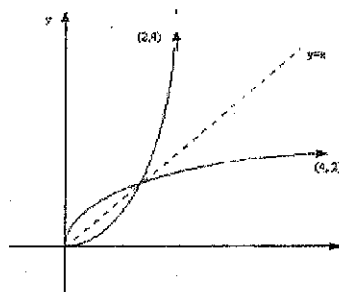
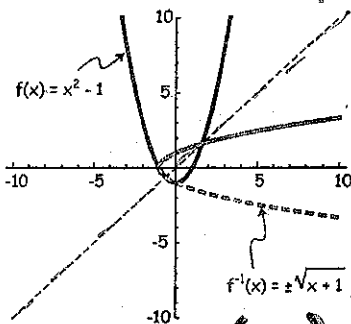
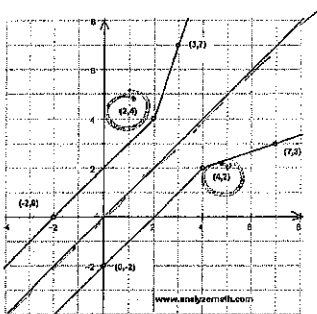
Yes, inverses

#### With a Graphing Calculator

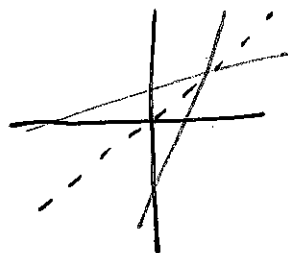
Option 2 - Graph 3 equations at once: the function, the inverse (you solved for),  $y=x$ .

\*If two functions are inverses, they are reflections of each other over the line  $y=x$ .

For example, they might look like the following:



Practice: Graph  $f(x) = 2x - 3$ ,  $g(x) = \frac{x+3}{2}$  and  $y=x$  to see if the two functions are inverses. Sketch what you see and state your answer clearly.



yes, inverses

**CLASSWORK: DO THESE ON A SEPARATE PIECE OF PAPER SHOWING WORK (and check your answers with BOB):**

p. 393 #5-11 all, 21-31 odd (For 10 and 11, you may use Option 1 or 2.)